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Optimization of Axial Flow Turbine for Mean-Line Design using Gentic Algorithm

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Abstract— In present study, Optimization is carried out for the meanline design of an axial flow turbine stage by considering aerodynamic performance, creep life and weight of the turbine stage. The mathematical formulation for the aerodynamic performance, creep life and weight for the turbine stage is brought out. The optimization is carried out using gentic algorithm in the present study. The fitness function is defined as a weighted average of parameters representing turbine isentropic efficiency, creep life and weight of the turbine. The design variables are flow coefficient, stage loading and turbine exit swirl angle with lower and upper bounds and the various constraints have been specified. The results of the study are presented in the paper.

Index Terms— Axial flow turbine, Mean line design, Optimization, Genetic algorithm, fitness function, turbine efficiency, Life, weight

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1 INTRODUCTION

The turbine design is a trade-off between aerodynamic, thermal, structural and manufacturing requirements. Hence, optimization plays an important role in the turbine design and development. The future trend in design optimization is to cover not only aerodynamic performance of turbines, but also creep life and weight of the turbine stage. Life cycle costs and product cycle time are additional, possible criteria during the design optimization process. Starting a new design, basic preliminary design steps calculate the mean dimensions of the machine.

The mean line design has the largest influence on the final product. The optimized mean line design greatly reduces the number of design iterations and the development time. The optimization methods, strategies for the axial flow turbine are given in [1], [2], [3], [4], [5]. In the mean line design, the meridional flow path, mean velocity triangles, number of vanes and blades are obtained by ensuring the required life and weight of the turbine stage. The results of the parametric study for the mean line design by considering the conflicting requirements of aerodynamic performance, creep life and weight are discussed in [6].

In general, if a function is defined on the basis of the design variables, depending upon the conditions of the efficiency, life and weight, the process of optimization, i.e the minimization or maximation, depending on the formulation of the fitness function, is to find out the right combination of the design variables, which will yield the best possible design in terms of aerodynamic performance, life and weight of the turbine module.

In the present study the optimization is carried out using the genetic algorithm. The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution.

The genetic algorithm is applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly non-linear. The genetic algorithm uses three main types of rules at each step to create the next generation from the current population: a) Selection rules select the individuals, called parents that contribute to the population at the next generation. b) Crossover rules combine two parents to form children for the next generation. c) Mutation rules apply random changes to individual parents to form children. The genetic algorithm differs from a classical, derivative-based, optimization algorithm in two main ways. Classical algorithms Generates a single point at each iteration, the sequence of points approaches an optimal solution and Selects the next point in the sequence by a deterministic computation whereas genetic algorithms Generates a population of points at each iteration, the best point in the population approaches an optimal solution and Selects the next population by computation which uses random number generators [7], [8], [9].

2 MEAN LINE DESIGN OF AXIAL FLOW TURBINES

A computer code is developed for the mean line design of an axial flow turbine to achieve high turbine efficiency by considering aerofoil cooling, castablity, structural aspects and weight. The Turbine mass flow (m), Inlet Total Pressure (P01), Inlet Total Temperature (T01), Power (P) to be produced, the target Total to total isentropic Efficiency(η) are obtained from the engine thermodynamic cycle. The rotational speed of the turbine is fixed by the compressor, because of the adverse pressure gradient in the compressor. The design variables for the mean line design are the flow coefficient(φ), the stage loading(ψ) and the turbine exit swirl angle(α_3). Besides these, pa-

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rameters like the Nozzle Guide vane and blade aspect ratio, Rotor Axial velocity ratio(Ca3/Ca2), NGV Axial velocity ratio(Ca1/Ca2), Blade speed ratio for Constant mean (KUm)/ biased design need to be specified based on the type of the turbine. The value of the Nozzle Guide vane loss coefficient is assumed initially.

Using the input parameters the meridional flow path, mean velocity triangles, stress function and reaction are calculated. The losses are estimated using the empirical correlations given in the references [10], [11],[12]. Incidence losses are estimated using the correlations given in the reference [13]. The optimum space-chord ratio and hence the number of blades are estimated using the correlation as explained in [10]. The corresponding aerofoil geometric characteristics are obtained based on the method explained in reference [10],[14].

The output contains the meridional flow path, number of blades and vanes, mean velocity triangles, blade and vane mean section geometric parameters, creep life and weight of the turbine stage.

The nomenaclature of the axial stations, the T-s diagram of the turbine stage and velocity triangle parameters are shown in Figures 1, 2 & 3 respectively.

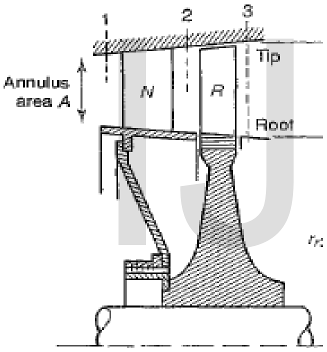

Fig.1. Nomenclature of Axial Stations

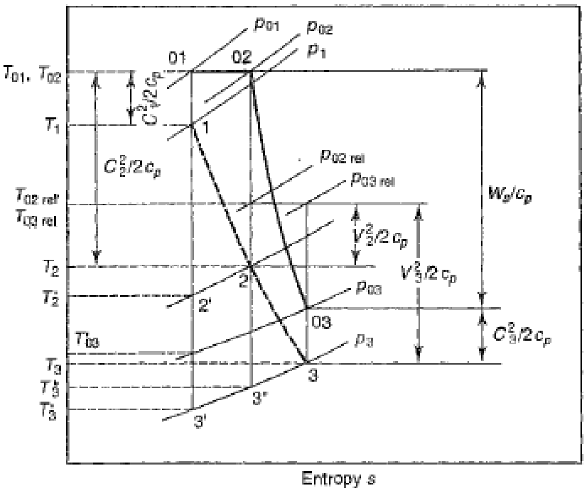

Fig.2. T-s diagram of Turbine stage

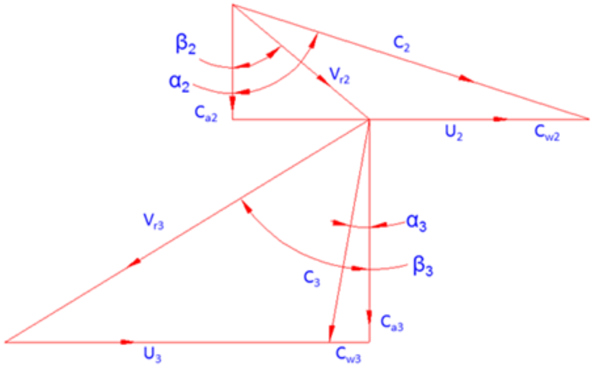

Fig.3. Parameters of the velocity triangle

3 MATHEMATICAL FORMULATION OF MEANLINE DESIGN

The mathematical formulation of the meanline design [15],[16],[17],[18] to evaluate the geometrical faeatures and turbine performance is given below.

$$P = mC_p\Delta T$$

$$C_p\Delta T = \frac{Power}{m}$$

$$\omega = \frac{2\pi N}{60}$$

Evaluation of the Mean Radius:

Hence,

$$\Psi = \frac{C_p\Delta T}{U^2}$$

$$U = \sqrt{\frac{C_p\Delta T}{\Psi}}$$

$$R_m = \frac{U}{\omega}$$

Evaluation of the Pressure Ratio:

$$\eta_{t-t} = \frac{\Delta T}{T_{01}\left(1-\left(\frac{1}{\frac{P_{01}}{P_{03}}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right)}$$

$$\frac{P_{01}}{P_{03}} = \left(\frac{1-\Delta T}{\eta T_{01}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$C_{a2} = U\Phi$$

$$C_{a3} = C_{a2}\left(\frac{C_{a3}}{C_{a2}}\right)$$

Calculations at Station 3

$$U_{m3} = UK_{U_m}$$

$$C_{\omega 3} = C_{a3} tan\alpha_3$$

$$C_3 = \left(\frac{C_{a3}}{cos\alpha_3}\right)$$

$$\mathrm{V_{r3}} = \sqrt{{C_{a3}}^2 + (U_{m3} + C_{\omega 3})^2}$$

$$\mathrm{P_3} = P_{03}\left(\frac{T_3}{T_{03}}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$\mathrm{a_3} = \sqrt{\gamma R T_3}$$

$$\mathrm{M_3} = \frac{C_3}{a_3}$$

$$\mathrm{M_{a3}} = \frac{C_{a3}}{a_3}$$

$$\mathrm{M_{r3}} = \frac{V_{r3}}{a_3}$$

$$\mathrm{T_{03rel}} = T_3\left[1+\left(\frac{\gamma-1}{2}\right)M{r_3}^2\right]$$

$$\frac{\mathrm{P_{03rel}}}{\mathrm{P_3}} = \left[1+\left(\frac{\gamma-1}{2}\right)M{r_3}^2\right]^{\frac{\gamma}{\gamma-1}}$$

Calculations at Station 2

$$\mathrm{U_{m2} = U}$$

$$mC_p\Delta T = m[U_{m2}C_{w2} + U_{m3}C_{w3}]$$

$$C_{w2} = \frac{[C_p\Delta \mathrm{T} - \mathrm{U_{m3}\,C_{w3}}]}{U_{m2}}$$

$$\alpha_2 = \tan^{-1}\left(\frac{C_{\omega 2}}{C_{a2}}\right)$$

$$\mathrm{C_2} = \frac{C_{a2}}{Cos\alpha_2}$$

$$\mathrm{V_{r3}} = \sqrt{{C_{a2}}^2 + (C_{w2} - U_{m2})^2}$$

$$\beta_2 = \tan^{-1}\left(\frac{C_{\omega 2} - U_{m2}}{C_{a2}}\right)$$

$$\mathrm{T_2} = \mathrm{T_{01}} - \frac{{C_2}^2}{2C_p}$$

$$\mathrm{a_2} = \sqrt{\gamma R T_2}$$

$$\mathrm{M}_2 = \frac{C_2}{a_2}$$

$$\mathrm{M}_{a2} = \frac{C_{a2}}{a_2}$$

$$\mathrm{M}_{r2} = \frac{V_{r2}}{a_2}$$

$$\mathrm{Y}_s = \frac{(P_{01} - P_{02})}{(P_{02} - P_2)}$$

$$\mathrm{P}_{02} = \frac{P_{01}}{\left[1 + Y_s\left(1 - \left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}\right)\right]}$$

$$\mathrm{P}_2 = \frac{P_{02}}{\left[1 + \left(\frac{\gamma - 1}{2}\right) Mr_2^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Areas at Station 2 and 3

$$\rho_2 = \frac{P_2}{(RT_2)}$$

Rotor inlet area,

$$\mathrm{A}_2 = \frac{m}{\rho_2 C_{a2}}$$

Rotor inlet blade height,

$$\mathrm{h}_2 = \frac{A_2}{2\pi R_m}$$

$$\rho_3 = \frac{P_3}{RT_3}$$

Rotor outlet area,

$$\mathrm{A}_3 = \frac{m}{\rho_3 C_{a3}}$$

Roter outlet blade height,

$$\mathrm{h}_3 = \frac{A_3}{2\pi R_m}$$

$$\mathrm{Y}_r = \frac{(P_{02rel} - P_{03rel})}{(P_{03rel} - P_3)}$$

$$\mathrm{C_x}R = \frac{(h_2 + h_3/2)}{h/C_x\ R}$$

$$\text{Rotor Hub Flare Angle} = \tan^{-1}\left\{\frac{\left[\left(R_{m3} - \frac{h_3}{2}\right) - \left(R_{m2} - \frac{h_2}{2}\right)\right]}{C_X R}\right\}$$

$$\text{Rotor Tip Flare Angle} = \tan^{-1}\left\{\frac{\left[\left(R_{m3} + \frac{h_3}{2}\right) - \left(R_{m2} + \frac{h_2}{2}\right)\right]}{C_X R}\right\}$$

Calculations at Station 1

$$\mathrm{C}_{a1} = C_{a2}\left(\frac{C_{a1}}{C_{a2}}\right)$$

$$\mathrm{C}_2 = \left(\frac{C_{a1}}{\cos\alpha_1}\right)$$

$$\mathrm{T}_1 = \mathrm{T}_{01} - \frac{{C_1}^2}{2C_p}$$

$$\mathrm{a}_1 = \sqrt{\gamma RT_1}$$

$$\mathrm{M}_1 = \frac{C_1}{a_1}$$

$$\mathrm{P}_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rho_1 = \frac{P_1}{RT_1}$$

Stator inlet area,

$$\mathrm{A}_1 = \frac{m}{\rho_1 C_{a1}}$$

Stator inlet blade height,

$$\mathrm{h}_1 = \frac{A_1}{2\pi R_m}$$

$$C_xN = \frac{(h_1 + h_2/2)}{h/C_x\,N}$$

$$\text{Stator Hub Flare Angle} = \tan^{-1}\left\{\frac{\left[\left(R_m - \frac{h_2}{2}\right) - \left(R_m - \frac{h_1}{2}\right)\right]}{C_X N}\right\}$$

$$\text{Stator Tip Flare Angle} = \tan^{-1}\left\{\frac{\left[\left(R_m + \frac{h_2}{2}\right) - \left(R_m + \frac{h_1}{2}\right)\right]}{C_X N}\right\}$$

$$\text{Zweifel Loading} = \ 2\left(\frac{s}{c}\right) Cos^2\alpha_2(\tan\alpha_2 + \tan\alpha_1)$$

With these mean geometrical and flow parameters, the losses are re-evaluated and turbine efficiency is re-calculated as below

$$\eta_s = \frac{1}{1 + \frac{1}{2}\phi\left[\frac{Y_R sec^2(\beta_3) + \left(\frac{T_3}{T_2}\right) Y_N sec^2(\alpha_2)}{\tan(\beta_3) + \tan(\alpha_2) - \frac{1}{\phi}}\right]}$$

4 MATHEMATICAL FORMULATION FOR CREEP LIFE

The life critical part in the turbine stage is the turbine rotor blade because of the centrifugal stresses and the blade metal temeperature. The method of evaluation of the creep life for the rotor blade is given below.The centrifugal stresses [15] are calculated from the relation

$$\sigma_c = \frac{2\pi K\rho A N^2}{3600}$$

where K is the blade tip to hub area taper ratio, which value is obtained from the designer's tacit knowledge.

Cooling effectiveness (η_c) value depends upon the cooling configuration adopted, Cooling air temperature (T_c) which depends upon the bleed extraction from the compressor. From this data, the metal temperature (T_{metal}) is calculated as shown below.

$$\eta_c = \frac{(T_{02rel} - T_{metal})}{(T_{02rel} - T_c)}$$

The value of the T_{metal} is calculated from the above relation, assuming a cooling effectiveness value of 0.45, which are typical of the Low Pressure turbines with impingement cooling, convective cooling and film-cooling.

Further the creep life is estimated from the Larson Miller parameter [19], Tmetal and σ_c shown in Figure 4 and the relation:

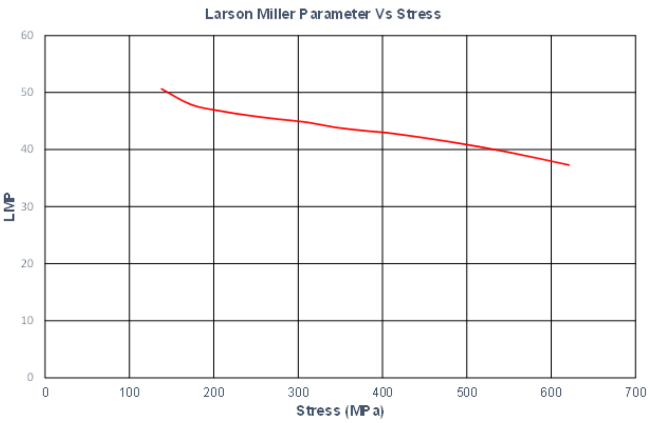

Fig. 4. Larson Miller parameter v/s Stress

$$LMP = (T + 460)(25 + logt) * 10^{-3}$$

5 MATHEMATICAL FORMULATION FOR TURBINE STAGE WEIGHT

The method to evaluate the weight of the turbine module [20],[21],[22] is given below.

The weight of the vanes are calculated from the relation

$$Vane\ Weight = C_1 * C * T_{max} * No.of\ Vanes * \left(\frac{h_1 + h_2}{2}\right) * \rho$$

where

$$C_1 = f(Stagger, \frac{LER}{C}, \frac{TER}{C}, Internal\ Configuration)$$

LER – Leading edge radius, TER – Trailing Edge Radius, C – Chord

The weight of the casings and the shroud segment is given by

$$Casing\ \&\ Shroud\ Segment\ Weight = C_2 * Tip\_axial\_length * 1.1 * r_{tip} * 2\pi * 2 * \rho_{718}$$

where C_2 is a numerical constant.

The weight of the blade is given by

$$Blade\ Weight = C_3 * C * T_{max} * No.of\ blades * \left(\frac{h_3 + h_2}{2}\right) * \rho$$

where

$$C_3 = f(Stagger, \frac{LER}{C}, \frac{TER}{C}, Internal\ Configuration)$$

Tmax – Aerofoil maximum thicknesss

The weight of the disc is given by

$$Disc\ Weight = C_4 * f(Blade\ Weight)$$

Where

$$C_4 = f(Blade\ fixing\ arrangement, Shafting\ arrangement)$$

The weight of the sealing arrangement of the vanes is given by

$$Vane\ Sealing\ Weight = f(LP\ Vane\ Hub\ axial\ length, No.\ of\ seals)$$

The total weight of the turbine module is given by

$$\begin{aligned} Total\ Weight = {} & Vane\ Weight + Casing\ \&\ Shroud\ Segment\ Weight \\ & + Vane\ Sealing\ Weight + Blade\ weight + Disc\ Assembly\ weight \end{aligned}$$

6 OPTIMIZATION OF THE MEAN LINE DESIGN

The genetic algorithm code [23] has been adopted for optimizing the turbine mean line design with the above formuation. The fitness function is a single objective function defined with different weightages, namely 50% for performance, 25% for life and 25% for weight of the turbine stage. The effect of change in weightages to (33.3%, 33.3%, 33.3%) are also explored. The fitness function which is formulated as described above is optimized with constraints on the design variables. The initial population size is chosen as 100 generations. The selection function is based on stochastic method. The elite count in reproduction is 2, and the crossover fraction is 0.8 and mutation fraction is 0.01. For convergence of the code, the function tolerance is taken as 1e-06. 0. The three independent design variables for the fitness function, namely Stage Loading / Mean radius, Flow coefficient and exit swirl angle with lower and upper bounds for each are defined in the code.

The variable / input vector is given by

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \psi \\ \phi \\ \alpha_3 \end{bmatrix} \quad (1)$$

The fitness function is defined by

$$f(X) = K_1(1-\eta) + K_2(Life) + K_3(Weight) \quad (2)$$

Where K_1, K_2 and K_3 are weight functions.

The constraints are: α2, β2, β3, M2, M2rel, M3rel, Reaction, flare angles, s/c of the blade and vane, AN^2, tip speed, blade metal temperature. In the mean line design, the station 1 referes to the stator inlet, station 2 refers to the rotor inlet and station 3 refers to the rotor outlet. The flow chart for the optimization is shown in Fig. 5.

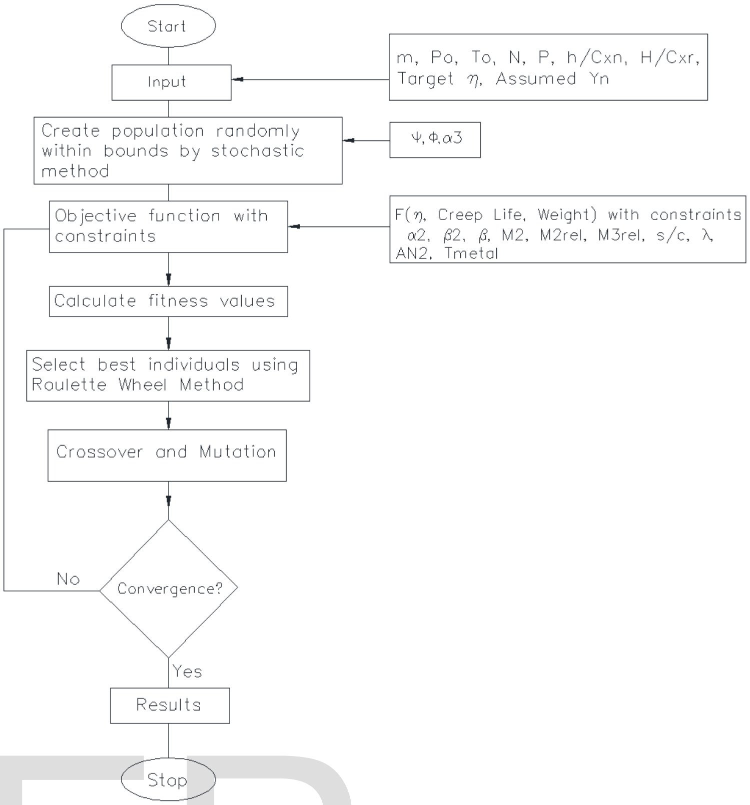

Fig. 5. Flow Chart of the Optimization process

7 RESULTS AND DISCUSSIONS

A parametric study has been carried out to study the effect of the design variables independently on efficiency, creep life and weight of the turbine stage. Optimized values of turbine efficiency, life and weight with respect to stage loading and flow coefficient and exit swirl angle are obtained for different combinations of the weight functions and imposed on the results obtained from the parametric study. The convergence plot of the optimization code showing the best fitness and mean fitness values are shown in Figure 6.

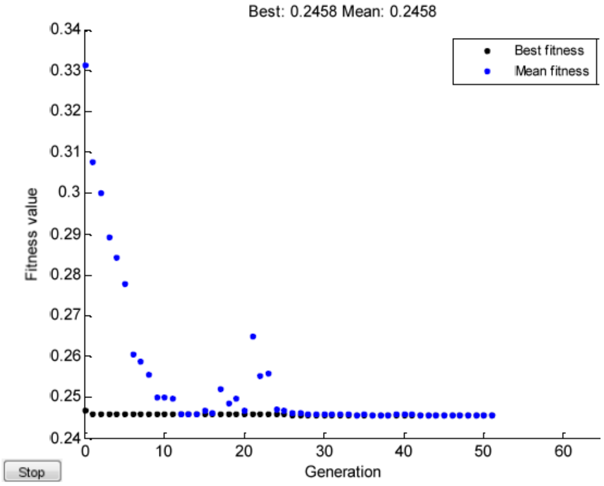

Fig. 6. Converegence Plot of the Opmitization tool

Figure 7 shows the optimized turbine efficiency superimposed on the parametric study results with respect to flow coefficient for different stage loadings at an optimal swirl angle. Similarly Figure 8 and 9 optimized turbine life and weight superimposed on the parametric study results with respect to flow coefficient for different stage loadings and swirl angle respectively. From the figures it can be observed that the optimum values represent an optimized or compromised solutions based on the trends obtained from the parametric study of the mean-line parameters. The optimized value of the flow coefficient and stage loading are 0.84 and 1.55 respectively at an exit swirl angle of 12 degrees, which is the upper bound considering the downstream component performance.

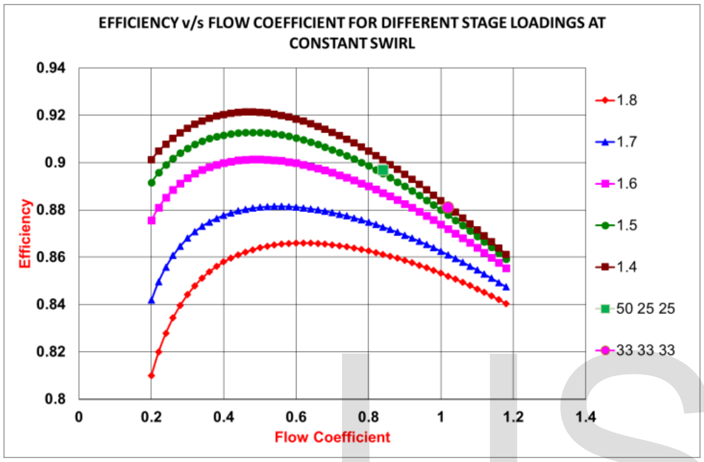

Fig. 7. Efficiency v/s Flow coefficient for differet stage loadings at constant swirl

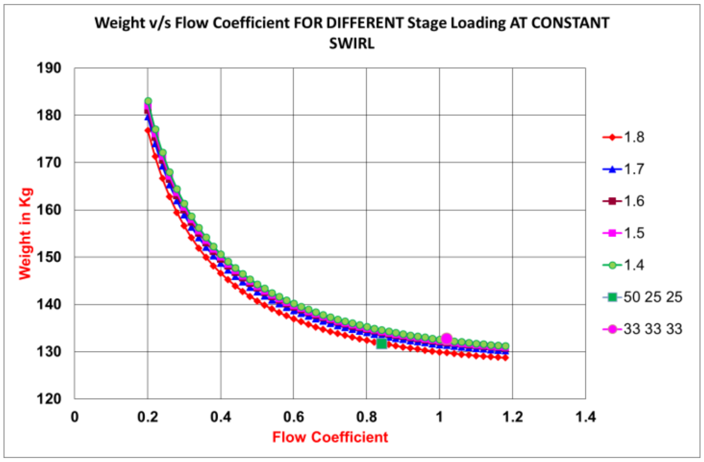

Fig. 8. Wieght v/s Flow Coeffciceint for different stage loadings

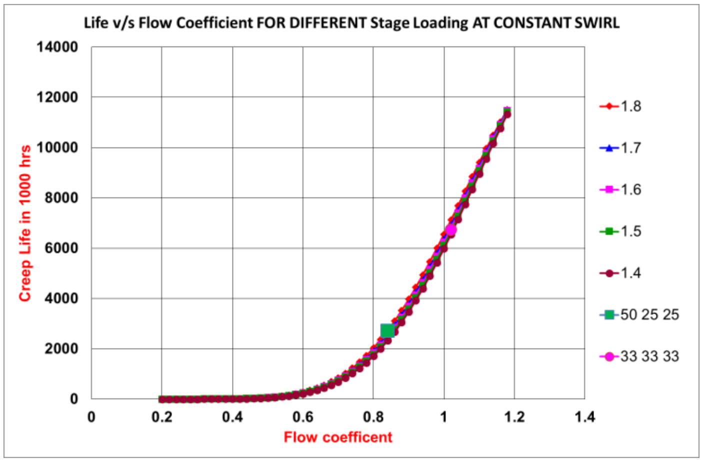

Fig. 9. Life v/s Flow Coefficient for different stage loadings

The optimized flow path and the mean velocity triangles obtained from the optimization of the mean line design are shown in figures 10 & 11

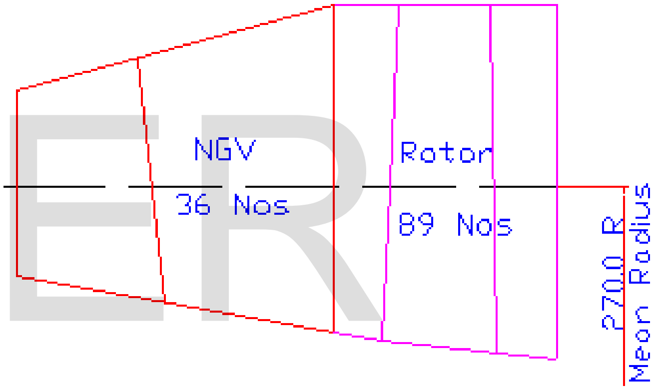

Fig. 10. Optimized Turbine flow path

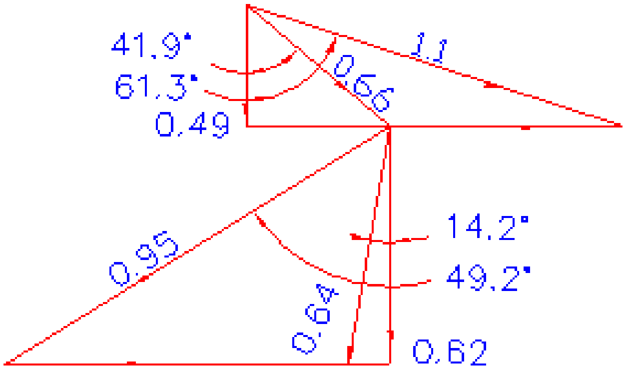

Fig. 11. Optimized Mean Velocity Triangle

8 CONCLUSION

The optimization of the mean-line design of a typical low pressure turbine of military aero-gas turbine engine is attempted by using genetic algorithm. The fitness function is formulated by considering the aerodynamic performance, weight and life of the turbine module. The optimized values for different weightages of performance, weight and life are presented in the paper.

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